Corollary of Abel's test

A uniformly convergent series $\sum u_n(x)$ remains uniformly convergent on $[a,b]$ if its terms are each multiplied by a function $a_n(x)$, $a \leq x \leq b$, provided that the sequence $[a_n(x)]$ is uniformly bdd on $[a,b]$ (i.e., $\exists$, $K > 0$, such that $|a_n(x)| \leq K$ for all x in $[a,b]$ and for all n). and monotonic in n, for each $x \in [a,b]$

Under the given condition $\{a_n(x)\}$ converges pointwise. Let us write for each $x \in [a,b]$.

$$b_n(x) = \begin{cases} \lim\limits_{n \to \infty} a_n(x) - a_n(x), \text{ or} \\ a_n(x) - \lim\limits_{n \to \infty} a_n(x) \end{cases}$$

according as $\{a_n(x)\}$ is monotonic increasing or decreasing. With this function $b_n(x)$, we ~~defin~~ deduce as above that the series $\sum b_n(x) \, u_n(x)$ converges uniformly on $[a,b]$.

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Also since $\sum u_n(x)$ and hence $\sum [\lim a_n(x)] u_n(x)$ converge uniformly on $[a,b]$. The uniform convergence of $\sum a_n(x) u_n(x)$ then follows easily.